Teaching multiplication facts – parental involvement and interventions

Andy Richards reports on a pilot study to improve early number skills

We all remember our teachers telling us how important it is to 'learn your tables'. Some children enjoy learning their multiplication facts by rote. However, for the bulk of children, this is onerous and therefore often very difficult for teachers and parents to promote. Multiplication facts are easy to teach, but often difficult to *learn*.

In the new mathematics curriculum, there are many opportunities for teachers to encourage children to look for short-cuts and alternative methodologies to learning multiplication facts by rote. Teaching the multiplication facts strategically helps children to make connections and reduces the burden on their memory. In an Early Years and Key Stage 1 setting, many children enjoy chanting and/or singing times tables. Following the curriculum; children learn 2×, 10×, 5× and the concepts of doubling and halving early in their school. Once they have moved onto the 3× and 4× tables, they can be reassured that, due to commutativity (supported by the mental images provided by arrays), they have covered aspects of the times tables that are often perceived to be more difficult.

Simple methods, like encouraging children to use the small number first (as opposed to the larger number in addition) can help enormously. The number of mental 'jumps' that a child has to make can be radically reduced. In addition, square numbers are taught at an earlier stage at Marlbrook Teaching School Alliance, Herefordshire – taking advantage of the striking visual images of the associated arrays – and act as further reference points when children are given a multiplication problem. Visual patterns can be also used to help children learn compensatory methods for the 9 times table (multiply by 10 and subtract the number being multiplied).

While there is no easy way to learn the 7× and 8× tables, by employing the methods above, the teacher can be assured that a child has the means to develop rapid recall of multiplication facts. So why is this so often not the case? Frequently, children arrive in upper Key Stage 2 with very poor knowledge of multiplication facts. Part of this may be due to multiplication not being a target for a cohort, or due to the reticence of some teachers to dedicate time for the teaching and learning of these facts within school time. Times tables are often viewed as the equivalent of spelling in Literacy – it is a homework task, to be supported by parents.

In these situations, there is often no alternative to learning multiplication by rote – a tedious and laborious task for any child. In this paper, I examine the efficacy of an established intervention technique, precision teaching, in developing children's times table knowledge.

What is precision teaching?

The key components of Precision Teaching are: to set time-based mastery criteria for each curriculum step, to provide daily opportunities for practice and timed measurement, to chart performance on a graph called the Standard Behaviour Chart and to change procedures when the chart shows they're not working (Pennypacker, Koenig and Lindsley, 1972; White and Haring, 1980). Precision Teaching is widely advocated to allow pupils to take responsibility for their own learning and be aware of the result of their daily practice.

A common reason for failure in numeracy is a lack in the acquisition of basic skills which hinders the progress smoothly to more advanced material. When they do not achieve sufficient levels of basic arithmetic computation, students usually experience difficulty learning more difficult aspects like long division, algebra and other advanced/more abstract maths skills. Thus, many so-called "learning disabil-ities" turn out to be no more than a failure of the schools to measure and to work toward fluency in basic skills. Precision Teachers have found that a few minutes per day of timed practice on carefully sequenced skills can often eliminate what were previously considered irremediable learning problems.

A number of Precision Teaching researchers, notably Kunzelmann and colleagues (Magliocca, L.A., Rinaldi, R.T., Crew, J.L., and Kunzezlmann, H.P., 1977), have worked to establish count per minute fluency standards for a wide range of academic skills. Using fluency standards and brief, timed assessment procedures, they've been able to identify students in need of special help with a higher degree of predictive validity, and greater cost-effectiveness than when using more traditional screening techniques. With regular (for example, monthly) one-minute timings on clusters of skills throughout entire schools and school systems, administrators and curriculum specialists have been able to track students' progress (and program effectiveness) across curriculum areas, classrooms, grade levels, and schools with a remarkable degree of precision and objectivity.

Many computer-based training lessons restrict the pace at which students can move form one response to another and common classroom teaching techniques provide such infrequent opportunities for individual responding that students are unlikely to maintain attention or to become fluent.

Precision teaching of multiplication facts

There are a number of ways that children can learn multiplication facts, but methods for ***teaching*** an understanding of these facts are relatively limited. Those children with a good grasp of doubling and halving and those able to make connections within the number system are able to extend simple facts to recall others. Those children who have a good grasp or suitable images of how multiplication works can be encouraged to use commutativity and compensation. There are also patterns, rhymes and a variety of tricks that can be learnt. However, many children have to resort to learning by rote or find this repetition to be the most efficient method. This is where the enjoyable games and fluency of Precision Teaching comes to the fore. Precision Teaching is cost effective; quick to undertake and allows children to take ownership of their learning.

One of the most empowering aspects of precision teaching is the way that results are recorded. To measure fluency, precision teaching utilizes a semi-logarithmic chart called a Standard Celeration Chart. This chart allows for demonstration of changes in rate of acquisition and allows the teacher to quickly assess a student's performance accelerating though time. By utilizing this chart, teachers are able to quickly adjust the curriculum to maximize the student's performance and learning. At the same time, the results of learning are magnified in the early parts of precision teaching, increasing the child's involvement and interest.

Precision Teaching and Active expression

Precision Teaching used with interactive technology allows the development of materials and procedures which free students to respond as rapidly and as often as they are able. The use of Active expression pods is a fun and easy way for children to respond to questions, in a timed setting. The data is recorded by associated software in a variety of formats and can be linked to an automated spreadsheet to analyse and graph pupil's results. It is easy then to present the results in a graphical form, or comparatively in the form of a league. Children really respond to this, especially when presented discretely on the basis of individual tables, with the final aim of competing on an 'all table' challenge.

Methodology

Nine children took part in the study. Children who had difficulties recalling number facts were selected from Years 4–6. In each year group; one child received no intervention apart from the normal diet of mental methods received in class, including daily multiplication tests; one child received an additional intervention at school while; the last child undertook precision teaching at school and at home. The school sessions were lead by teaching assistants, while home sessions were lead by parents/carers. All adults received the same training before the onset of the programme and used the same resources and methods to train the children in fluency and to record their progress. An initial test was undertaken to establish the children's prior knowledge of multiplication facts. The children were required to learn 2×, 5×, and 10× multiplication facts. These tables are the first to be taught in the current curriculum, and they also form the basis of some written and mental methods for multiplication and division in the school's calculation progression.

Discussion

The celeration charts (figure 1) show that, while there were some differences, all children had poor fluency prior to the study. The rates of progression shown the show a few discernible patterns:

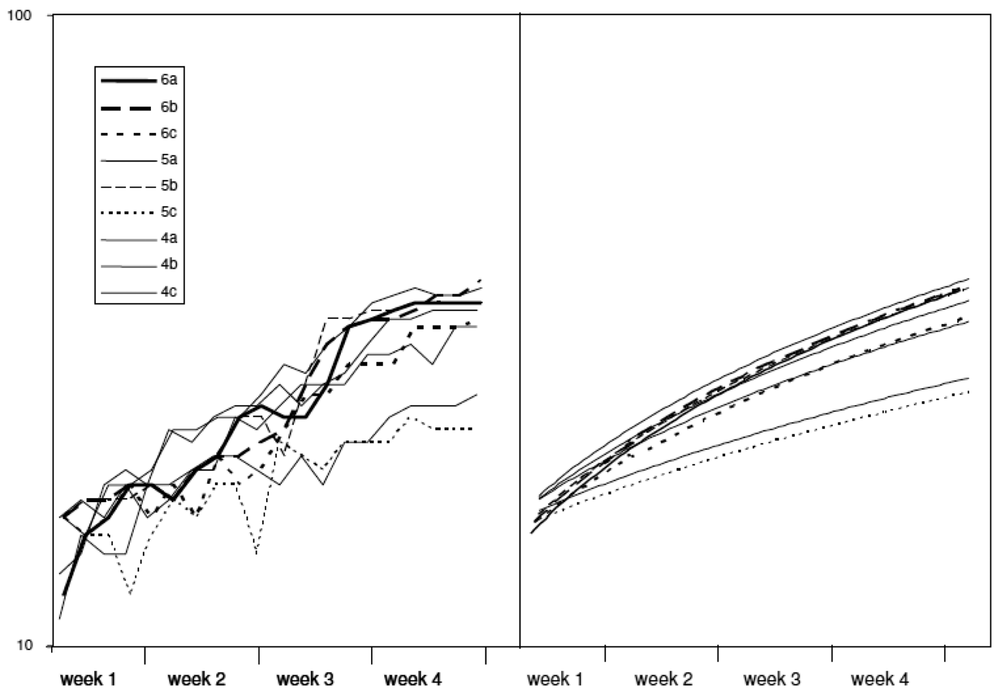

Figure 1

- Children who received no intervention, progressed at a steady, but slower rate than those children receiving intervention.
- There are less marked differences between the rate of progression in children receiving intervention at school and those who also repeated the intervention at home.
- While rates of progress are similar, the stability of progress in groups a and b, appear to be affected by the omission of practice at the weekend, where children's progress dips or plateaus early the following week.
- Children who have undertaken intervention start to peak shortly after week 3 and development slows, or plateaus during week 4. Conversely, children who have received no intervention, show less stability in their progress and appear to need more practice in order to reach their potential.
- The correlation matrix (figure 2) shows that the patterns of development are very similar in children who have received intervention, while, the stability of progress in children who did not take part in precision teaching suggest that they found it harder to commit facts to memory and suffered from sudden dips in performance

The differences in the results suggest two main focuses for further research; the role of a home-school programme, and the value and methods used in regular testing.

The 'stability' displayed in the home-school programme is an important factor, as neuroscience and research into metacognition suggests repetition, rather than dropping facts that we already know, is an important part of the learning process.

Tests are traditionally used to assess what students have learned with the assumption that the test itself is relatively neutral. However, research in cognitive psychology shows that tests not only measure knowledge but enhance it (Roediger and Karpicke, 2006). The testing effect refers to the fact that material that is recalled or recognized is better retained on future tests than material studied for an equivalent amount of time. This finding has prompted research into how the type of test given; the spacing of tests, the timing and types of feedback after tests; promote retention, as well as how these variables can be combined to optimally enhance learning in different domains. Repeated testing at home, as well as the activities, may have provided more stability.

An additional point to bear in mind both for future studies and considering the recent emphasis on rote learning, is evidence that normal preparation for tests has been brought into question by those researching the value of metacognition. When revising for tests, we often test ourselves and then repeat the cycle, dropping items that we got right. This is principally to make studying (and testing) quicker and allows us to focus our effort on the things we haven't yet learnt. It's a plan that seems to

	6a	6b	6c	5a	5b	5c	4a	4b	4c
6a	1	0.969004	0.953415	0.977785	0.964987	0.870577	0.966073	0.95483	0.902166
6b	0.969004	1	0.981772	0.964227	0.982437	0.863681	0.925973	0.957409	0.846178
6c	0.953415	0.981772	1	0.963324	0.950288	0.884002	0.933673	0.951107	0.843591
5a	0.977785	0.964227	0.963324	1	0.954397	0.910786	0.977798	0.96171	0.892532
5b	0.964987	0.982437	0.950288	0.954397	1	0.844027	0.909469	0.948853	0.832696
5c	0.870577	0.863681	0.884002	0.910786	0.844027	1	0.893276	0.881643	0.871646
4a	0.966073	0.925973	0.933673	0.977798	0.909469	0.893276	1	0.93847	0.927483
4b	0.95483	0.957409	0.951107	0.96171	0.948853	0.881643	0.93847	1	0.854723
4c	0.902166	0.846178	0.843591	0.892532	0.832696	0.871646	0.927483	0.854723	1

Figure 2

make perfect sense, but as a plan it is disastrous if we really want to learn properly, as research suggests that the recall of known facts, as well as the ones that we commonly get wrong, are important to the process (Roediggerand Karpicke, 2006; Karpicke and Roediggerr, 2007; McDaniel *et al.*, 2007). Simple testing offers benefits in itself- getting arithmetical questions wrong actually increases the plasticity of the brain and increases it's ability to learn (Boaler, 2013). There's more to testing than finding out what students know – tests can also help us learn and remember.

The involvement of parents/ carers in learning number facts appears to have had a positive effect, although a larger data-set is required to test this. In addition to the benefits of more activities and testing that the home-school programme allowed, there is a huge volume of work that suggests other factors may be important in this respect. According to Every Child Matters, (2003) "Research suggests that parenting appears to be the most important factor associated with educational achievement at age 10, which in turn is strongly associated with achievement in later life. Parental involvement in education seems to be a more important influence than poverty, school environment and the influence of peers." Desforges (2003) has demonstrated a large body of evidence which points to the link between a parent's involvement in a child's learning and a child's subsequent achievement. Researchers report that parent participation in their children's schooling frequently enhances children's self-esteem as well as their children's academic achievement as parents are actively providing encouragement and support for their children's learning (Epstein, 1996, Safran, 1996). Current research suggests that a parent's involvement may go beyond this. It is not only that parent's are important for providing aspirations, Norwegian data indicate that an additional year of either mother's or father's education increases their children's education by as little as one-tenth of a year (Ermisch and Pronzato, 2010). Other causal factors may be social or gender based. Boaler (1997, 2013) feels that parents play a huge role in engendering a 'mindset' that encourages enquiry and enthusiasm, particularly for girls.

Therefore, this study prompts a number of questions. Is precision teaching the best method for learning and testing rapid recall? If it is, can precision teaching be modified in the light of recent research in metacognition and neuroscience to make it a more powerful tool? Can the home- school model used here be scaled up, both to incorporate other areas of learning and to reach more children? How can we teach the recall of number facts in an effective way, while empowering children and parents/carers and tightening academic links with school? Finally, does precision teaching, and allied rote learning techniques, encourage the 'linked' thinking that is so vital in mathematics and, if not, how do we modify these techniques to encourage such a skill? Such attributes are very difficult to measure or quantify, but have a huge bearing on a student's success in mathematics post-16.

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