WORKING TOWARDS A 'CHILD FRIENDLY' WRITTEN PROGRESSION FOR DIVISION

Andrew Richards documents how one school modified its approach to division

If you were to ask any primary school child which aspects of numeracy they found difficult, it is likely that they would refer to division, fractions, decimals, percentages, and/or ratio and proportion. Each is related, and many children with mathematical difficulties have failed to build up adequate representations – verbal, visual, and abstract- before being asked to use and understand associated formal and informal written methods.

Some of the written algorithms associated with division are contentious; Ofsted have questioned the validity of 'standard' written methods in a number of documents (Ofsted, 1993; 2002; 2008; 2011). As a consequence of these issues, there has been a large amount of debate regarding alternative methods that might be employed in the teaching and learning of division and associated areas (*i.e.* Anghileri *et al.*, 2002; Bradford, 2011; Thompson, 2005; 2011; 2012)

Analysis of pupil performance in assessment over an 18-month period, illustrates that the areas outlined above are problematic for children in Years 4-6 at Marlbrook. In order to address this issue, the school refined the methods used to support the development of effective, but personalised, mental strategies and has endeavoured to encourage formal and informal written methods that draw on these strategies.

This article does not intend to advocate a whole-scale change in the way that children are taught, but to illustrate how evidence-based practice (*cf* Thompson, 2012) and the use of more personalised, informal written methods, can help pupils to access calculations that they find difficult using prescriptive, 'standard' methods.

Rationale

Two main written methods are employed in Key stage 2 for division: *short multiplication*, or the '*bus stop'* method, and '*chunking*', or the '*partial quotients method*'; the use of the divisor, or multiples thereof, in repeated subtraction. With addition, subtraction, and multiplication there is a gradual progression towards more formal written methods. These methods can be linked with mental strategies at every stage. This is not the case with division where, in general, the dominant mental strategy builds up the quotient - result of division - through repeated addition of multiples, or *chunks* of multiples. When visualising division, most people would elect to chunk *towards* rather than *away* from the dividend, the number being divided. 'Subtractive' chunking uses subtraction and is, therefore, counterintuitive to many children. A further consequence of using this method is that children are only allowed to draw on multiplication facts *if* the dividend is a multiple of the divisor, that is with no remainder. If the dividend is not a multiple of the divisor, the process does not draw on, or apply, any knowledge of multiples.

Chunking is widely purported (*i.e*. Campbell, 2005; Eastaway, 2010*)* to be preferable to 'long division' as the learner uses a single procedure established with smaller numbers that can be made more efficient by the user. Chunking also allows for differentiation and progression in a more personalised way. Chunking is more holistic, focusing throughout on the whole calculation and its meaning, rather than just rules for generating successive digits. As a result, it requires genuine understanding to be successful, rather than just the ability to follow a ritualised procedure. However, children need to have an established, ideally compact, method for written subtraction before using chunking efficiently. It becomes very complicated for a child who relies on a number-line, or the expanded method for subtraction. As a result, the differentiation that chunking allows means that the less able have to do more calculations, an operation that lower abilities are likely to be less proficient at in the first place.

Ofsted (2011) has reported that most of the more successful schools, together with the bulk of specialised/private schools, allow more numerate learners to move onto the 'bus stop' method. However, children can employ the short division algorithm without understanding how it works. The higher abilities can execute the methods by rote, but do not employ an adapted version of it as a mental strategy. It is difficult to give meaning to the procedure, or provide supporting concrete examples, models, and images. This process is symbolic, and requires a different thought process from chunking. Children '*just have to learn it'*. To '*understand*' it, their knowledge of place value has to be water-tight.

One school's approach to 'personalised' division methods

Thompson (2005; 2012a, 2012b) and Bradford (2011) have both discussed the possibilities of moving away from standardised methods for division and have advocated methods which are more in tune with children's understanding of the operation. Marlbrook School has used these ideas as a starting point to refine the school's written method for calculation, supported by a shift in emphasis within

the teaching and learning of mental strategies.

Strategies employed in the early stages of the progression, employed by children working at, or towards, Level 1, emphasise the use of concrete equipment and drawings.

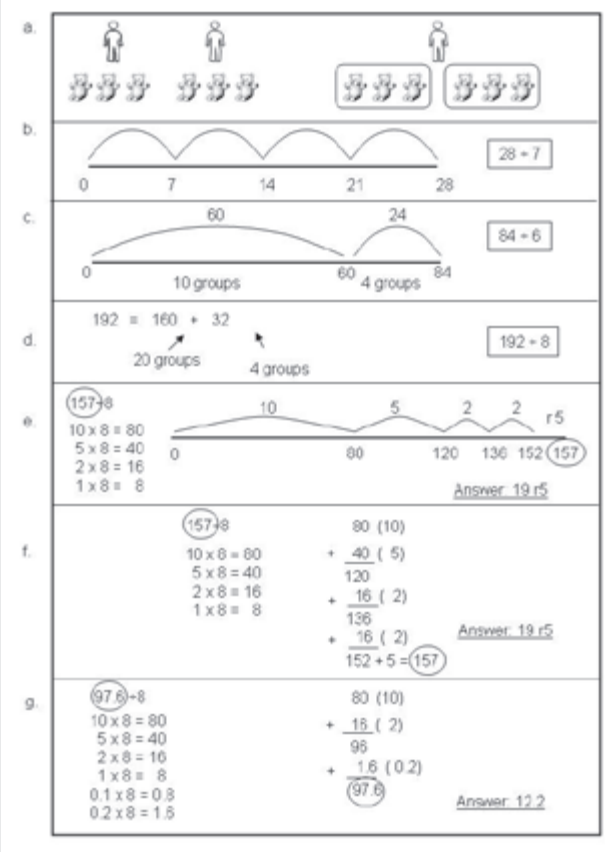


Figure 1

Later, dots or tally marks can either be shared out one at a time or split up into groups- thus covering both important aspects at an early stage. In addition, at this stage, children are encouraged to use, or visualise, arrays of numbers or number-lines/ Cuisenaire rods are also promoted, depending on what the learner prefers. Formerly, the next stage of the progression involved developing the children's use of either: short division; repeated subtraction, or subtractive chunking. The progression has been modified to promote extended use of the number-line and the development of an expanded, informal written method, see Figure 1e,f, and g.

The use of a number-line, initially by using the divisor, and later by using bigger 'chunks' or multiples of the divisor is used to encourage a deeper understanding of the process of division, see Figure 1. Obviously, this does not cover the understanding of division as 'grouping' and sharing, but, through deconstructing the number line as a means of checking the calculation, caters for both images. The first stage of the process involves children defining four multiplication facts: 1x, 2x, 5x and 10x the divisor. These are the first multiplication facts that children learn as part of the national framework, and draws on knowledge of doubling and halving. The fact that these multiples form the basis of our monetary system also provides an experiential aspect. The teaching of an informal method is derived from this, but at the child's own pace. 'Chunking Up', or complementary multiplication, draws on these simple multiplication facts and addition.

However, when a learner has a sound grasp of division, usually illustrated by the speed of their mental methods, they are encouraged to explore modified informal methods or to use the 'standard' methods that are promoted in Key stage 3. Commonly, children use short division for single unit divisors, and 'chunking' when dividing by two-digit numbers. Higher abilities may even use short division for the latter. Providing children have a good grasp of what happens when we multiply and /or divide by 10, 100, and 1000, the methods outlined above can be used to divide decimals, or provide decimal remainders, see Figure 1g. While this method is not as straightforward as when using the 'bus stop' method for decimals, it promotes a fuller understanding of place value. This progression is easily supported by concrete materials and images to consolidate children's understanding. Number-lines, number-tracks and Cuisenaire rods are the most useful practical resources in this respect and support images, in either a linear form, or as an array.

Therefore, the principal method for division throughout this progression involves chunking of the dividend towards, rather than away from the divisor. The following section looks at the initial impact of this change on children working at, or below, national expectations in upper Key stage 2.

Evidence

Changes in the progression were assessed by analysing children's progress over a six week period. Children were given a range of division sums to solve, derived from past Standard Assessment Tests. No input was given to the children, other than coverage of pre-learning vocabulary and associated mental methods. Learners working at different levels were then assessed after they had focussed on the new elements of the written progression for a 3- to 4-day period, dependent on pupils attainment. Obviously, one would hope that children's understanding and accuracy would improve following this period(!), but in order to evaluate the impact of the new progression, the pupils were assessed with levelled questions, and the results were compared to their assessed level in the Number strand.

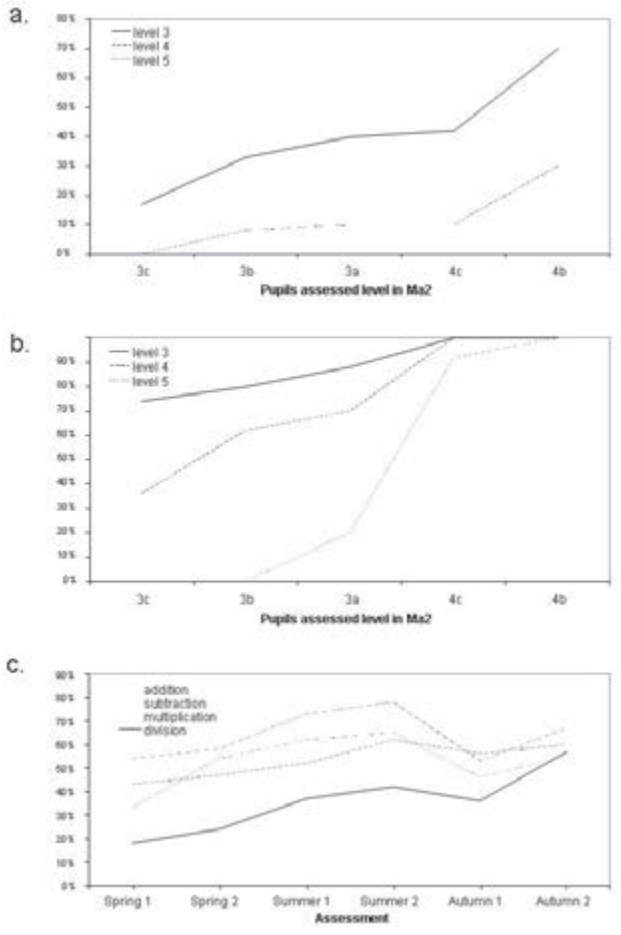
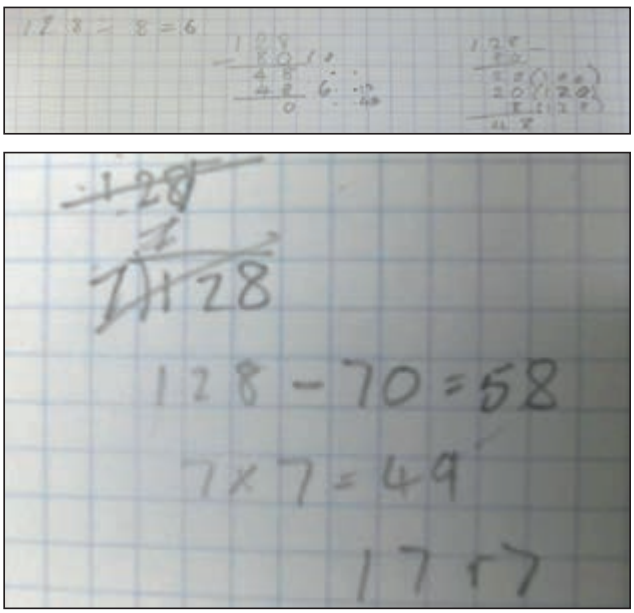


Figure 2

Before being 'taught', the written methods used were assessed against division sums at Levels 3-5 (TU÷U, HTU÷U and HTU÷TU). Children chose a variety of strategies. Most chose personalised jottings, utilising a number-line. For the higher level questions, most pupils chose to utilise subtractive methods; either the bus-stop method, or subtractive chunking.



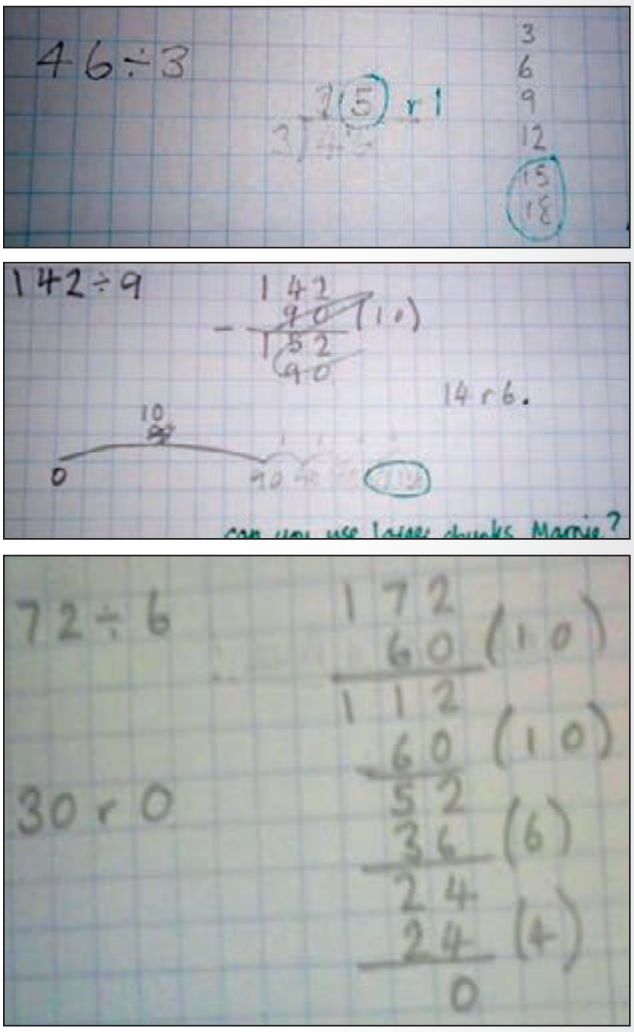


Figure 3

Some 14 out of 30 children did not attempt the Level 4 or 5 questions. Those pupils employing a number-line enjoyed more success than those that used a standard method. The problems that most learners encountered can be grouped into four categories: inadequate knowledge of number facts; poor use of decomposition for the subtractive aspect of chunking; the lack of a systematic informal approach, or; the failure to recall an appropriate written method.

Following teaching and learning using the new written progression, levels of attainment improved. Many children who had struggled with 'Level 3 type' questions were now using additive strategies to successfully solve sums characteristic of Level 5 calculation. The bulk of lower ability pupils employed a partial multiplication facts table and a number-line, while some mastered the more formal written format, and could relate this to their preferred mental strategies. Many of the children working at Level 4, chose to use the written format, initially supported by the partial multiplication facts table.

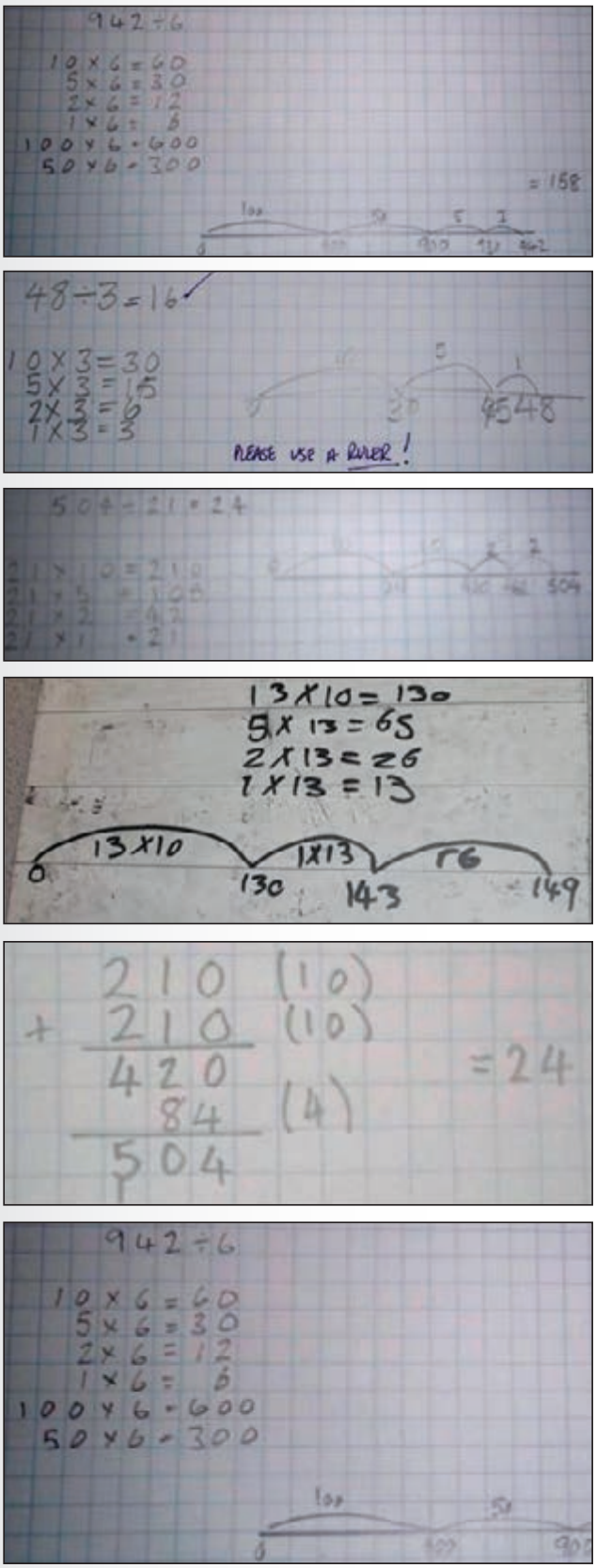


Figure 4

Once confident, children within this group could employ the written method without the support of written number facts.

Obviously, the longer-term implications of using this progression on pupil attainment remain to be seen. Initial qualitative and quantitative, see Figure 2, results are promising and suggest that the proposed informal methods provide children with the opportunities to modify methods that reflect their own mental strategies.

Conclusions

The principal advantage of additive division towards, rather than away from, the dividend is that it is linked to mental methods, and therefore offers the best opportunity to consolidate learner understanding. It fits naturally into a progression and allows children to fully exercise known facts, divisibility rules, and place value. In addition, links between division and multiplication are more explicit and it is particularly useful for those children that find decomposition problematic. The standard written methods do not allow differentiation beyond the number of digits in the divisor or dividend, whereas additive division can be deconstructed for different levels of understanding, and may be personalised by the user.

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QR Code

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